us 1 Name________________________

ed Rates- Trigonometry Date________________________

5-foot ladder is leaning against a wall. The bottom of the ladder is being pulled a
the wall at a rate of 3 feet per second. How fast is the angle between the ladder
ound changing at the instant the base of the ladder is 15 feet from the wall?

$\frac{dx}{dt} = 3$ ft/sec

$\frac{d\theta}{dt} = ?$ when $x=15$

$\cos\theta = \frac{x}{25}$

$-\sin\theta \frac{d\theta}{dt} = \frac{1}{25}\frac{dx}{dt}$

$\overset{\cos^{-1}}{\cancel{\cos}}\,\theta = \overset{\cos^{-1}}{\frac{15}{25}}$

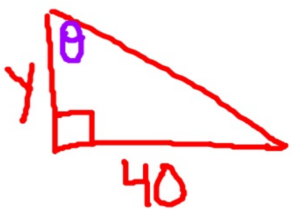
$\theta = \cos^{-1}(15/25)$

$\theta = .927$ radians

$-\sin(.927)\frac{d\theta}{dt} = \frac{1}{25}\cdot 3$

$\frac{d\theta}{dt} = -.15$ rad/sec

\ balloon is rising vertically at a rate of 2 ft per second. The horizontal distance
balloon and an observer on the ground is 40 feet. How fast is the angle between
otenuse and the vertical distance changing after 50 seconds?



$\frac{dy}{dt} = 2 \text{ ft/sec}$

$\frac{d\theta}{dt} = ?$ when $y = 100$ ft.

$\tan\theta = \frac{40}{100}$

$\theta = .381$ rad

$\tan\theta = \frac{40}{y}$

$\tan\theta = 40y^{-1}$

$\sec^2\theta \frac{d\theta}{dt} = -40y^{-2}\frac{dy}{dt}$

$\sec^2(.381)\frac{d\theta}{dt} = \frac{-40}{100^2} \cdot 2$

$\sec^2(.381)\frac{d\theta}{dt} = -.008$

$\frac{1}{(\cos(.381))^2}$

$\boxed{\frac{d\theta}{dt} = -.007 \text{ rad/sec}}$

woman on a dock is pulling in rope fastened to the bow of a small boat. If the wor
; are 10 feet higher than the point where the rope is attached to the boat and if
eving the rope at a rate of 2 ft per second, how fast is the angle where the rope
hed to the boat changing when 25 feet of rope is still out?

$\sin\theta = \frac{10}{25}$

$\theta = .412 \text{ rad.}$

$\frac{dr}{dt} = -2 \text{ ft/sec}$

$\frac{d\theta}{dt} = ?$ when $r = 25$

$\sin\theta = \frac{10}{r}$

$\sin\theta = 10r^{-1}$

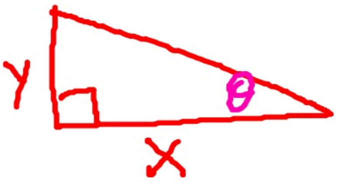
$\cos\theta \frac{d\theta}{dt} = -10r^{-2}\frac{dr}{dt}$

$\cos(.412)\frac{d\theta}{dt} = \frac{-10}{25^2} \cdot -2$

$\cos(.412)\frac{d\theta}{dt} = .032$

$\boxed{\frac{d\theta}{dt} = .035 \text{ rad/sec}}$

a given instant, the legs of a right triangle are 5 cm and 12 cm long. If the shor
asing at a rate of 1.5 cm/sec and the long leg is decreasing at a rate of 2.2 cm/se
is the angle between the long leg and the hypotenuse changing?



$\frac{dy}{dt} = 1.5 \text{ cm/sec}$ $\frac{d\theta}{dt} = ?$ When $y = 5$

$\frac{dx}{dt} = -2.2 \text{ cm/sec}$ $x = 12$

$\tan\theta = \frac{5}{12}$

$\theta = .395 \text{ rad}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = y \cdot x^{-1}$

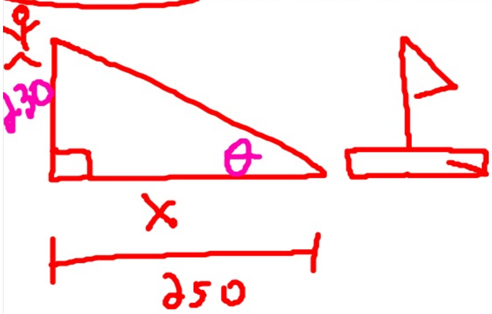
$\sec^2\theta \frac{d\theta}{dt} = \frac{dy}{dt} \cdot x^{-1} + y(-x^{-2})\frac{dx}{dt}$

$\sec^2(.395) \cdot \frac{d\theta}{dt} = \frac{1.5 \cdot 12}{12 \cdot 12} - \frac{5}{12^2} \cdot (-2.2)$

$\sec^2(.395)\frac{d\theta}{dt} = \frac{29}{144}$

$\boxed{\frac{d\theta}{dt} = .172 \text{ rad/se}}$

shepherd is on top of a cliff, 230 feet above a lake. A boat is approaching the bo
e cliff at a rate of 20 feet per second. The boat started at 250 ft away from th
10 seconds, how fast is the angle of elevation changing?



$$\frac{dx}{dt} = -20 \text{ ft/sec}$$

$$\frac{d\theta}{dt} = ? \text{ when } x = 50$$

$$\tan\theta = \frac{230}{50}$$

$$\theta = 1.357 \text{ rad}$$

$$\tan\theta = \frac{230}{x}$$

$$\tan\theta = 230x^{-1}$$

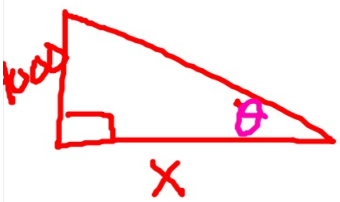
$$\sec^2\theta \frac{d\theta}{dt} = -230x^{-2}\frac{dx}{dt}$$

$$\sec^2(1.357)\cdot\frac{d\theta}{dt} = -\frac{230}{50^2}\cdot(-20)$$

$$\sec^2(1.357)\frac{d\theta}{dt} = 1.84$$

$$\boxed{\frac{d\theta}{dt} = .083 \text{ rad/sec}}$$

plane is flying at an elevation of 4000 feet. A radar station on the ground is tra
plane. The angle of elevation from the radar station to the plane is increasing at
10 radian per second. At the instant the angle of elevation is $\frac{1}{2}$ radian, how fast
zontal distance changing?



$$\frac{d\theta}{dt} = .1 \text{ rad/sec}$$

$$\frac{dx}{dt} = \text{ when } \theta = .5$$

$$\tan(.5) = \frac{4000}{x}$$

$$x = 4000/\tan(.5)$$

$$x = 7321.951$$

$$\tan\theta = \frac{4000}{x}$$

$$\tan\theta = 4000x^{-1}$$

$$\sec^2\theta \frac{d\theta}{dt} = -4000x^{-2}\frac{dx}{dt}$$

$$\sec^2(.5)\cdot(.1) = \frac{-4000}{(7321.951)^2}\frac{dx}{dt}$$

$$\boxed{} \div \left(-4000 \div (7321.951)^2\right)$$

$$\frac{dx}{dt} = -1740.274 \text{ ft/sec}$$